Moment Curvature of Circular Section

Input

Column

Diameter: $D := 1000mm$

Clearance to Reinforcement: $clr := 50mm$

Longitudinal Reinforcment

Number of Bars: $n_{bars} := 10$

Cross Sectional Area: $As_l := 1006mm^2$

Diameter of Bar: $db_l := 35.8mm$

Transverse Bars

Cross Sectional Area: $As_t := 284mm^2$

Diameter Bar: $db_t := 19.1mm$

Spacing of Hoops or Spirals: $s := 100mm$

Type of Confinement: $trans_{bar} := hoop$

Materials

Steel - A706

Yield Stress $f_y := 475MPa$

Ultimate Stress $f_u := 655MPa$

Modulus of Elasticity: $E_s := 200000MPa$

Yield Strain $\varepsilon_y := \frac{f_y}{E_s}$

Axial Strain @ On-set of Hardening $\varepsilon_{sh} := 0.0115$

Ultimate Strain $\varepsilon_{su} := .12$

Concrete

Compressive Strength of Concrete $f'_c := 35.0MPa$

Modulus of Elasticity $E_c := 28000MPa$

Spalling Strain $\varepsilon_{sp} := 0.005$

Longitudinal Strain a f 'c $\varepsilon_{co} := 0.002$

Axial Load

$P := 100$

Analysis

$n_{strips} := 20$ Number of strips to "break" 1/2 the circle into

Bar Reinforcement Layout

Number of Bars: $n_{bars} = 10$

Radius To Center of Bars $r := \frac{D}{2} - clr - db_t - \frac{db_l}{2}$ $r = 413\,mm$

Angle Between Bars: $delta := \frac{2\cdot\pi}{n_{bars}}$ $delta = 0.628$

Center of Circle: $center\_x := \frac{D}{2}$ $center\_x = 0.5\,m$

$center\_y := center\_x$ $center\_y = 0.5\,m$

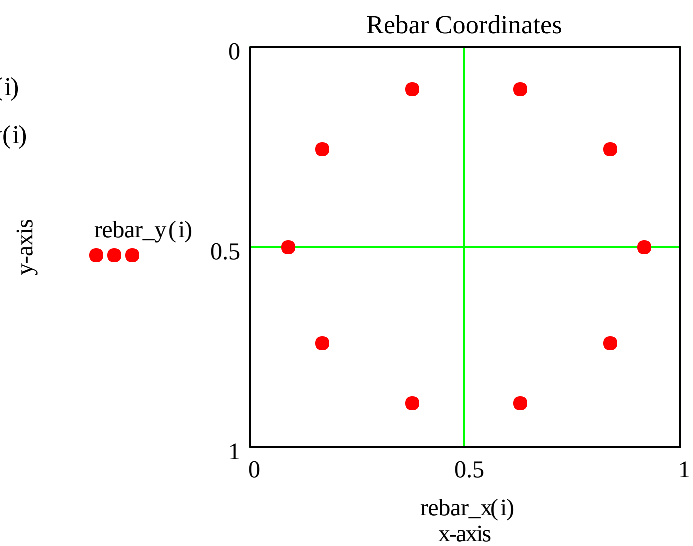
$$rebar\_x(i) := \left| \begin{array}{l} angle \leftarrow delta \cdot (i) \\ x \leftarrow \cos(angle) \cdot r + center\_x \\ x \end{array} \right.$$

$$rebar\_y(i) := \left| \begin{array}{l} angle \leftarrow delta \cdot (i) \\ y \leftarrow \sin(angle) \cdot r + center\_y \\ y \end{array} \right.$$

$i := 0 .. n_{bars} - 1$

$resteel\_x_i := rebar\_x(i)$

$resteel\_y_i := rebar\_y(i)$



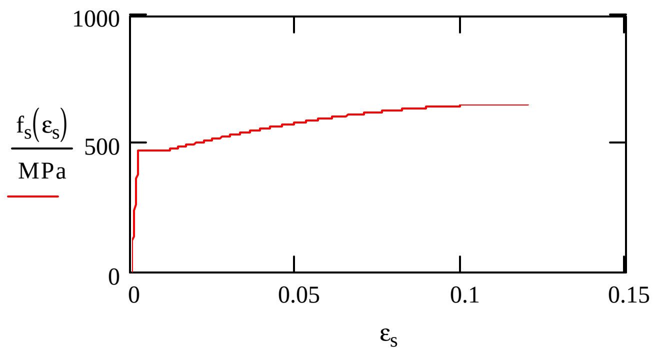
Park's Steel Model (Modified)

$r := \varepsilon_{su} - \varepsilon_{sh}$

$$m := \frac{\frac{f_u}{f_y} \cdot (30 \cdot r + 1)^2 - 60 \cdot r - 1}{15 \cdot r^2}$$

$$f_s(\varepsilon_s) := \left[\begin{array}{l} \text{sign} \leftarrow 1 \\ \text{if } \ \varepsilon_s < 0.0 \\ \quad \left| \begin{array}{l} \varepsilon_s \leftarrow |\varepsilon_s| \\ \text{sign} \leftarrow -1 \end{array}\right. \\ \text{sign}\cdot E_s \cdot \varepsilon_s \ \ \text{if } \ \varepsilon_s \le \varepsilon_y \\ \text{sign}\cdot f_y \ \ \text{if } \ \varepsilon_y < \varepsilon_s \le \varepsilon_{sh} \\ \text{sign}\cdot\left[f_u - \left[(f_u - f_y)\cdot\left(\frac{\varepsilon_{su} - \varepsilon_s}{\varepsilon_{su} - \varepsilon_{sh}}\right)^2\right]\right] \ \ \text{if } \ \varepsilon_{sh} < \varepsilon_s \le \varepsilon_{su} \\ 0.0 \ \ \text{if } \ \varepsilon_s > \varepsilon_{su} \end{array}\right]$$

$$\varepsilon_s := 0, 0.0001 .. \varepsilon_{su}$$



Mander's Unconfined Concrete Model

$$Esec := \frac{f'_c}{\varepsilon_{co}} \qquad Esec = 1.75 \times 10^4 \, MPa$$

$$r := \frac{E_c}{E_c - Esec} \qquad r = 2.667$$

$$fc1(\varepsilon_c) := \left| \begin{array}{l} x \leftarrow \dfrac{\varepsilon_c}{\varepsilon_{co}} \\ \dfrac{f'_c \cdot x \cdot r}{r - 1 + x^r} \end{array} \right.$$

$$fc2(\varepsilon_c) := \left| \begin{array}{l} x \leftarrow \dfrac{\varepsilon_c}{\varepsilon_{co}} \\ f'_c \cdot \left(\dfrac{2 \cdot r}{r - 1 + 2^r} \right) \cdot \left(1 - \dfrac{\varepsilon_c - 2 \cdot \varepsilon_{co}}{\varepsilon_{sp} - 2 \cdot \varepsilon_{co}} \right) \end{array} \right.$$

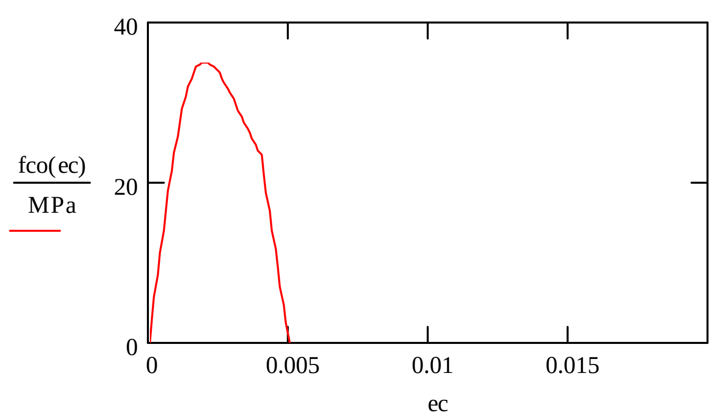
$$fc3(ec) := 0 \cdot MPa$$

$$fco(strain) := \left| \begin{array}{ll} fc1(strain) & \text{if } 0.0 < strain \le 2 \cdot \varepsilon_{co} \\ fc2(strain) & \text{if } 2 \cdot \varepsilon_{co} < strain \le \varepsilon_{sp} \\ fc3(strain) & \text{otherwise} \end{array} \right.$$

$$ec := 0, 0.0001 .. 0.005$$

$fco(ec) =$

0	MPa
2.799	
5.593	
8.368	
11.109	
13.795	
16.403	
18.91	



Mander's Confined Concrete Model

Total Area of Main Reinforcement: $A_s := n_{bars} \cdot As_l$

Core Diameter (cl of confining steel) $ds := D - 2(clr + db_l)$

Clear Distance Between Spirals or Hoops: $sc := s - db_t$

$\rho_s := \frac{4 \cdot As_t}{s \cdot ds}$ $\qquad \rho_s = 0.014$

$\rho_{cc} := \frac{4 \cdot A_s}{\pi \cdot ds^2}$ $\qquad \rho_{cc} = 0.019$

$$ke := \begin{cases} \dfrac{1 - 0.5 \cdot \dfrac{sc}{ds}}{1 - \rho_{cc}} & \text{if spiral} \\[2ex] \dfrac{\left(1 - 0.5 \cdot \dfrac{sc}{ds}\right)^2}{1 - \rho_{cc}} & \text{otherwise} \end{cases} \qquad ke = 0.922$$

$fl := 0.5 \cdot ke \cdot \rho_s \cdot f_y$ $\qquad fl = 3.003\,\text{MPa}$

Confined Concrete Strength

$$f'_{cc} := f'_c \cdot \left(2.254 \cdot \sqrt{1 + \frac{7.94 \cdot fl}{f'_c}} - 2 \cdot \frac{fl}{f'_c} - 1.254\right) \qquad f'_{cc} = 52.393\,\text{MPa}$$

$$\varepsilon_{cc} := \varepsilon_{co} \cdot \left[5 \cdot \left(\frac{f'_{cc}}{f'_c} - 1\right) + 1\right] \qquad \varepsilon_{cc} = 0.007$$

$$Esec := \frac{f'_{cc}}{\varepsilon_{cc}} \qquad Esec = 7.517 \times 10^3\,\text{MPa}$$

Bridge Automation

9/21/2003

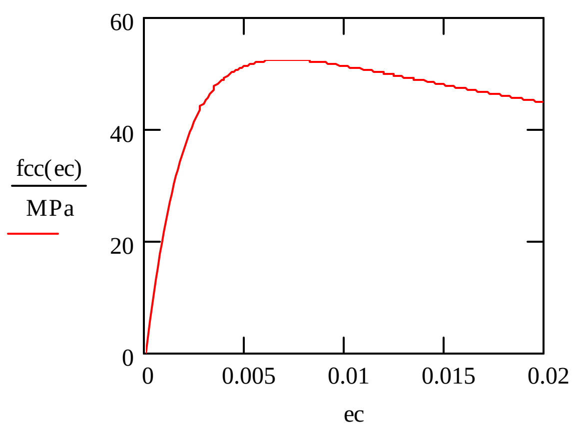
$$r := \frac{E_c}{E_c - Esec} \qquad r = 1.367$$

$$fcc(strain) := \begin{vmatrix} x \leftarrow \dfrac{strain}{\varepsilon_{cc}} \\ \dfrac{f'_{cc} \cdot x \cdot r}{r - 1 + x^r} \quad \text{if } strain \geq 0.0 \\ 0.0 \cdot MPa \quad \text{otherwise} \end{vmatrix}$$

Ultimate Concrete Strain: $\varepsilon_{cu} := 0.004 + 0.14 \cdot \rho_s \cdot \dfrac{f_y}{f'_{cc}}$ $\qquad \varepsilon_{cu} = 0.02141$

Limit of Ultimate Strain: $\varepsilon_{cu} := \text{if}(\varepsilon_{cu} > 0.020, 0.020, \varepsilon_{cu})$ $\qquad \varepsilon_{cu} = 0.02000$

$$ec := 0.0000, 0.0001 .. \varepsilon_{cu}$$



Plane Strain

$$e_{bot}(na, e_{top}) := e_{top} - e_{top} \cdot \frac{D}{na}$$

$$e(y, e_{top}, e_{bot}) := e_{top} - (e_{top} - e_{bot}) \cdot \frac{y}{D}$$

Discretize Circular Shape

Sector Area

$$area\_sector(d, y) := \frac{d^2}{4} \cdot acos\left(1 - \frac{2 \cdot y}{d}\right) - \left(\frac{d}{2} - y\right) \cdot \sqrt{d \cdot y - y^2}$$ y <= r

r - radius of sector
y - distance from extreme fiber on top of section (+axis downward)

Sector Strip Area

$$area\_slice(d, y_1, y_2) := area\_sector(d, y_2) - area\_sector(d, y_1)$$ y2 > y1

Height of Concrete Core Strips $$core_{inc} := \frac{0.5(D - 2clr)}{n_{strips}}$$

Height of Concrete Cover Strips $$ext_{inc} := \frac{0.5D}{n_{strips}}$$

Area of Core $$A_{core}(i) := area\_slice(D - 2clr, core_{inc} \cdot i, core_{inc} \cdot i + core_{inc})$$

Area of Gross Section $$A_{ext}(i) := area\_slice(D, ext_{inc} \cdot i, ext_{inc} \cdot i + ext_{inc})$$

CG of Core Strips $$y_{core}(i) := clr + \frac{core_{inc}}{2} + core_{inc} \cdot i$$

CG of Gross Section Strips $$y_{ext}(i) := \frac{ext_{inc}}{2} + ext_{inc} \cdot i$$

Solution

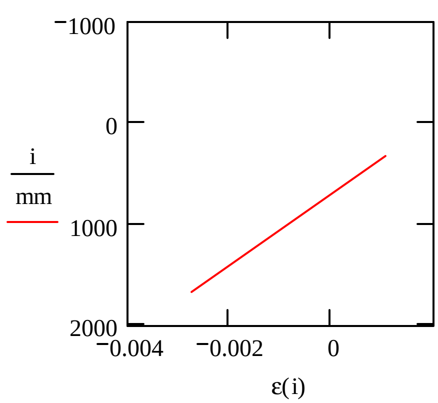
Check First Yield Point

Extreme Rebar Location	$y_s := \max(resteel\_y)$	$y_s = 893\,mm$
Extreme Core Concrete Fiber	$y_{cc} := clr + \frac{db_t}{2}$	$y_{cc} = 60\,mm$
Neutral Axis	$na := 270mm$	
Strain in Extreme Concrete Fiber	$\varepsilon_{top} := .00102$	

$$\varepsilon(y) := \varepsilon_{top} - \frac{y}{na}\cdot\varepsilon_{top}$$

Strain in Extreme Rebar	$\varepsilon_{bot} := \varepsilon(y_s)$	$\varepsilon_{bot} = -0.002353$

$i := 0..\, D$



$i := 0..\, n_{bars} - 1$

Steel Compression

$$C_s := \sum_i As_I \cdot \left| \begin{array}{l} value \leftarrow f_s(\varepsilon(rebar\_y(i))) \\ value \ \text{ if } \ (value \ge 1.0 \cdot MPa) \\ 0.0MPa \ \text{ otherwise} \end{array} \right.$$

$C_s = 267\,kN$

Steel Tension

$$T_s := \sum_i As_I \cdot \left| \begin{array}{l} value \leftarrow f_s(\varepsilon(rebar\_y(i))) \\ value \ \text{ if } \ value \le 0.0MPa \\ 0.0MPa \ \text{ otherwise} \end{array} \right.$$

$T_s = -2015\,kN$

Steel Void Compression

$$C_{s\_void} := \sum_i As_I \cdot \left| \begin{array}{l} value \leftarrow fcc(\varepsilon(rebar\_y(i))) \\ value \ \text{ if } \ value \ge 0.0 \cdot MPa \\ 0.0MPa \ \text{ otherwise} \end{array} \right.$$

$C_{s\_void} = 34\,kN$

Concrete Compression Minus Steel Voids

$$F_{cover}(i) := \left(A_{ext}(i) \cdot fco\left(\varepsilon\left(y_{ext}(i)\right)\right)\right)$$

$$F_{core}(i) := A_{core}(i) \cdot fcc\left(\varepsilon\left(y_{core}(i)\right)\right)$$

$$F_{void}(i) := A_{core}(i) \cdot fco\left(\varepsilon\left(y_{core}(i)\right)\right)$$

$$C_c := \left[\sum_{i=0}^{(n_{strips}-1)} \left(F_{cover}(i) + F_{core}(i) - F_{void}(i)\right)\right] - C_{s\_void}$$

$C_c = 1860\,kN$

Moment Steel Compression

$$M_{cs} := \sum_i \left(\frac{D}{2} - rebar\_y(i)\right) As_I \cdot \left| \begin{array}{l} value \leftarrow f_s(\varepsilon(rebar\_y(i))) \\ value \;\; \text{if} \;\; (value \geq 1.0 \cdot MPa) \\ 0.0MPa \;\; \text{otherwise} \end{array} \right.$$

$M_{cs} = 102\,m\,kN$

Moment Steel Tension

$$M_{ts} := \sum_i \left(\frac{D}{2} - rebar\_y(i)\right) As_I \cdot \left| \begin{array}{l} value \leftarrow f_s(\varepsilon(rebar\_y(i))) \\ value \;\; \text{if} \;\; value \leq 0.0MPa \\ 0.0MPa \;\; \text{otherwise} \end{array} \right.$$

$M_{ts} = 546\,m\,kN$

Moment Steel Void

$$M_{s\_void} := \sum_i \left(\frac{D}{2} - rebar\_y(i)\right) As_I \cdot \left| \begin{array}{l} value \leftarrow fcc(\varepsilon(rebar\_y(i))) \\ value \;\; \text{if} \;\; value \geq 0.0 \cdot MPa \\ 0.0MPa \;\; \text{otherwise} \end{array} \right.$$

$M_{s\_void} = 13\,m\,kN$

Moment Concrete Compression

$$M_{cc} := \sum_{i=0}^{(n_{strips}-1)} \left[F_{cover}(i) \cdot \left(\frac{D}{2} - y_{ext}(i)\right) + F_{core}(i) \cdot \left(\frac{D}{2} - y_{core}(i)\right) - F_{void}(i) \cdot \left(\frac{D}{2} - y_{core}(i)\right)\right]$$

$M_y := M_{cs} + M_{ts} + M_{s\_void} + M_{cc}$ $\qquad$ $M_y = 1391\,m\,kN$

First Yield Moment Results

$\phi_y := \frac{\varepsilon_{top}}{na}$ $\qquad$ $\phi_y = 0.00378\,m^{-1}\,rad$ $\qquad$ Curvature

$M_y = 1391\,m\,kN$ $\qquad$ Moment from program 1373kN m

$\left(1 - \frac{1373}{1391}\right) = 1.3\,\%$ $\qquad$ Percent difference