

## Pile Foundation

### Materials

Rebar Yield Strength  $f_y := 420\text{MPa}$

Concrete Strength  $f_c := 25\text{MPa}$

Concrete Density  $\rho_{\text{soil}} := 20 \frac{\text{kN}}{\text{m}^3}$

Soil Density  $\rho_{\text{conc}} := 25 \frac{\text{kN}}{\text{m}^3}$

### Footing Dimensions

Width (longitudinal)  $w_{\text{ftg}} := 10\text{m}$

Length (transverse)  $l_{\text{ftg}} := 10\text{m}$

Depth  $d_{\text{ftg}} := 2.5\text{m}$

### Piles

Number of Piles  $n := 8$

Y Offset  $y_{\text{offset}} := \frac{1}{2} \cdot w_{\text{ftg}} - 700\text{mm}$   $y_{\text{offset}} = 4.3\text{m}$

X Offset  $x_{\text{offset}} := \frac{1}{2} \cdot l_{\text{ftg}} - 700\text{mm}$   $x_{\text{offset}} = 4.3\text{m}$

### Column Dimensions

Width (longitudinal)  $w_{\text{col}} := 1000\text{mm}$

Length (transverse)  $l_{\text{col}} := 2000\text{mm}$

Area  $\text{area}_{\text{col}} := w_{\text{col}} \cdot l_{\text{col}}$

### Soil Cover

Depth  $d_{\text{soil}} := 150\text{mm}$

### Reinforcement Clearances

Top Clearance  $clr_{top} := 75\text{mm}$

Bottom Clearance  $clr_{bot} := 150\text{mm}$

### Loads

Axial := 100000kN

$M_{xx} := 100000\text{kN}\cdot\text{m}$

$M_{yy} := 100000\text{kN}\cdot\text{m}$

### Calculations

$$I_{xx} := y_{offset}^2 \cdot n \qquad I_{xx} = 1.479 \times 10^8 \text{ mm}^2$$

$$I_{yy} := x_{offset}^2 \cdot n \qquad I_{yy} = 1.479 \times 10^8 \text{ mm}^2$$

### Axial Load Due to Column Load + Soil + Footing

$$P := \text{Axial} + (l_{ftg} \cdot w_{ftg} \cdot d_{ftg}) \cdot \rho_{conc} + (l_{ftg} \cdot w_{ftg} - \text{area}_{col}) \cdot d_{soil} \cdot \rho_{soil}$$

$$P = 106544 \text{ kN}$$

### Maximum and Minimum Pile Reactions

$$\frac{P}{n} + \frac{M_{yy} \cdot x_{offset}}{I_{yy}} + \frac{M_{xx} \cdot y_{offset}}{I_{xx}} = 19132 \text{ kN}$$

$$\frac{P}{n} - \frac{M_{yy} \cdot x_{offset}}{I_{yy}} - \frac{M_{xx} \cdot y_{offset}}{I_{xx}} = 7504 \text{ kN}$$

### Effective Column Dimensions

$$w_{coff} := \sqrt{\text{area}_{col} \cdot \frac{l_{col}}{w_{col}} \cdot \frac{w_{col}}{l_{col}}} \qquad w_{coff} = 1 \text{ m}$$

$$l_{coff} := \sqrt{\text{area}_{col} \cdot \frac{l_{col}}{w_{col}}} \qquad l_{coff} = 2 \text{ m}$$

## Cap Reinforcement

### Bottom Reinforcement X-X

$$Pile_c := \frac{P}{n} + \frac{M_{xx} \cdot y_{offset}}{I_{xx}} \quad Pile_c = 16225 \text{ kN} \quad M_{xx} = 100000000 \text{ N}\cdot\text{m}$$

### Critical Section

$$cs := \frac{1}{2} w_{coff} \quad cs = 0.5 \text{ m}$$

$$y_{cs} := \frac{1}{2} w_{ftg} - cs$$

$$M_{ftg} := -\frac{1}{2} y_{cs}^2 \cdot l_{ftg} \cdot d_{ftg} \cdot \rho_{conc} \quad M_{ftg} = -6328 \text{ kN}\cdot\text{m}$$

$$M_{soil} := -\frac{1}{2} y_{cs}^2 \cdot l_{ftg} \cdot d_{soil} \cdot \rho_{soil} \quad M_{soil} = -304 \text{ kN}\cdot\text{m}$$

$$M_{pile} := 3 \cdot Pile_c \cdot (y_{offset} - cs) \quad M_{pile} = 184965 \text{ kN}\cdot\text{m}$$

$$M_u := M_{pile} + M_{ftg} + M_{soil} \quad M_u = 178333 \text{ kN}\cdot\text{m}$$

$$\phi := 0.90$$

### Solver Initial Guess

$$A_s := 10000 \cdot \text{mm}^2$$

$$a := 1000 \text{ mm}$$

$$b := l_{ftg}$$

$$d := d_{ftg} - cl_{bot} \quad d = 2.35 \text{ m} \quad \text{Effective Depth}$$

Given

$$a = \frac{A_s \cdot f_y}{0.85 \cdot f_c \cdot b}$$

$$M_u = \left[ A_s \cdot f_y \cdot \left( d - \frac{a}{2} \right) \right] \cdot \phi$$

$$\text{result} := \text{Find}(A_s, a)$$

$$A_s := \text{result}_0 \quad A_s = 221364 \text{ mm}^2 \quad \leftarrow \text{Controls}$$

$$a := \text{result}_1 \quad a = 437.52 \text{ mm}$$

### Temperature Requirements

$$\frac{1}{8} \cdot \text{in}^2 \cdot \frac{l_{ftg}}{ft} = 2646 \text{ mm}^2$$

### Minimum Reinforcement - 1.2 Mcr

$$S := \frac{l_{ftg} \cdot d_{ftg}^2}{12}$$

$$f_r := 7.5 \cdot \sqrt{f'_c \cdot \text{psi}}$$

$$M_u := 1.2 \cdot S \cdot f_r \quad M_u = 1.946 \times 10^4 \text{ kN}\cdot\text{m}$$

Given

$$a = \frac{A_s \cdot f_y}{0.85 \cdot f'_c \cdot b}$$

$$M_u = \left[ A_s \cdot f_y \cdot \left( d - \frac{a}{2} \right) \right] \cdot \phi$$

result := Find( $A_s, a$ ) ← Solve for Area of Steel, and dimension a

$$A_s := \text{result}_0 \quad A_s = 22114 \text{ mm}^2$$

$$a := \text{result}_1 \quad a = 43.708 \text{ mm}$$

### Top Reinforcement X-X

$$\text{Pile}_t := \frac{P}{n} - \frac{M_{xx} \cdot y_{\text{offset}}}{I_{xx}}$$

$$\text{Pile}_t = 10411 \text{ kN} \quad \text{Compression Not Tension}$$

$$cs := \frac{1}{2} w_{\text{coffeff}}$$

$$cs = 0.5 \text{ m}$$

$$y_{cs} := \frac{1}{2} w_{ftg} - cs$$

$$M_{ftg} := -\frac{1}{2} y_{cs}^2 \cdot l_{ftg} \cdot d_{ftg} \cdot \rho_{conc} \quad M_{ftg} = -6328 \text{ kN}\cdot\text{m}$$

$$M_{soil} := -\frac{1}{2} y_{cs}^2 \cdot l_{ftg} \cdot d_{soil} \cdot \rho_{soil} \quad M_{soil} = -304 \text{ kN}\cdot\text{m}$$

$$M_{pile} := 3 \cdot Pile_t \cdot (y_{offset} - cs) \quad M_{pile} = 118686 \text{ kN}\cdot\text{m}$$

$$M_u := M_{pile} + M_{ftg} + M_{soil} \quad M_u = 112054 \text{ kN}\cdot\text{m}$$

$$M_u := \text{if}(M_u > 0.0 \text{ kN}\cdot\text{m}, 0.0 \text{ kN}\cdot\text{m}, M_u)$$

$$\phi := 0.9$$

#### Solver Initial Guess

$$A_s := 10000 \cdot \text{mm}^2$$

$$a := 1000 \text{ mm}$$

$$b := w_{ftg}$$

$$d := d_{ftg} - cl_{ftop}$$

Given

$$a = \frac{A_s \cdot f_y}{0.85 \cdot f_c \cdot b}$$

$$M_u = \left[ A_s \cdot f_y \cdot \left( d - \frac{a}{2} \right) \right] \cdot \phi$$

$$\text{result} := \text{Find}(A_s, a) \quad \leftarrow \text{~~~~~ Solve for Area of Steel, and dimension } a$$

$$A_s := \text{result}_0 \quad A_s = 0 \text{ mm}^2$$

$$a := \text{result}_1 \quad a = -9.802 \times 10^{-4} \text{ mm}$$

### Temperature Requirements

$$\frac{1}{8} \cdot \text{in}^2 \cdot \frac{w_{ftg}}{\text{ft}} = 2646 \text{ mm}^2$$

### Minimum Reinforcement - 1.2 Mcr

$$A_s := 1000 \text{ mm}^2 \quad \text{Guess}$$

$$S := \frac{w_{ftg} \cdot d_{ftg}^2}{12}$$

$$f_r := 7.5 \cdot \sqrt{f_c \cdot \text{psi}}$$

$$M_u := 1.2 \cdot S \cdot f_r \quad M_u = 1.946 \times 10^4 \text{ kN}\cdot\text{m}$$

Given

$$a = \frac{A_s \cdot f_y}{0.85 \cdot f_c \cdot b}$$

$$M_u = \left[ A_s \cdot f_y \cdot \left( d - \frac{a}{2} \right) \right] \cdot \phi$$

result := Find( $A_s, a$ ) <~~~~~ Solve for Area of Steel, and dimension a

$$A_s := \text{result}_0 \quad A_s = 21418 \text{ mm}^2 \quad <~~~ \text{Controls}$$

$$a := \text{result}_1 \quad a = 42.332 \text{ mm}$$

### Bottom Reinforcement Y-Y

$$\text{Pile}_c := \frac{P}{n} + \frac{M_{yy} \cdot x_{\text{offset}}}{I_{yy}} \quad \text{Pile}_c = 16225 \text{ kN} \quad M_{yy} = 100000000 \text{ N}\cdot\text{m}$$

$$cs := \frac{1}{2} l_{\text{coff}} \quad cs = 1 \text{ m} \quad l_{\text{col}} = 2 \text{ m}$$

$$x_{cs} := \frac{1}{2} l_{ftg} - cs$$

$$M_{ftg} := -\frac{1}{2} x_{cs}^2 \cdot w_{ftg} \cdot d_{ftg} \cdot \rho_{conc}$$

$$M_{ftg} = -5000 \text{ kN}\cdot\text{m}$$

$$M_{soil} := -\frac{1}{2} x_{cs}^2 \cdot w_{ftg} \cdot d_{soil} \cdot \rho_{soil}$$

$$M_{soil} = -240 \text{ kN}\cdot\text{m}$$

$$M_{pile} := 3 \cdot Pile_c \cdot (x_{offset} - cs)$$

$$M_{pile} = 160627 \text{ kN}\cdot\text{m}$$

$$M_u := M_{pile} + M_{ftg} + M_{soil}$$

$$M_u = 155387 \text{ kN}\cdot\text{m}$$

$$\phi := 0.9$$

### Solver Initial Guess

$$A_s := 10000 \cdot \text{mm}^2$$

$$a := 1000 \text{ mm}$$

$$b := w_{ftg}$$

$$d := d_{ftg} - cl_{bot}$$

Given

$$a = \frac{A_s \cdot f_y}{0.85 \cdot f_c \cdot b}$$

$$M_u = \left[ A_s \cdot f_y \cdot \left( d - \frac{a}{2} \right) \right] \cdot \phi$$

$$\text{result} := \text{Find}(A_s, a)$$

$$A_s := \text{result}_0$$

$$A_s = 190128 \text{ mm}^2$$

<~~~~~ Controls

$$a := \text{result}_1$$

$$a = 375.782 \text{ mm}$$

### Temperature Requirements

$$\frac{1}{8} \cdot \text{in}^2 \cdot \frac{w_{ftg}}{\text{ft}} = 2646 \text{ mm}^2$$

Minimum Reinforcement - 1.2 Mcr

$$S := \frac{w_{ftg} \cdot d_{ftg}^2}{12}$$

$$f_r := 7.5 \cdot \sqrt{f_c \cdot \text{psi}}$$

$$M_u := 1.2 \cdot S \cdot f_r \quad M_u = 1.946 \times 10^4 \text{ kN}\cdot\text{m}$$

Given

$$a = \frac{A_s \cdot f_y}{0.85 \cdot f_c \cdot b}$$

$$M_u = \left[ A_s \cdot f_y \cdot \left( d - \frac{a}{2} \right) \right] \cdot \phi$$

result := Find( $A_s, a$ ) <~~~~~ Solve for Area of Steel, and dimension a

$$A_s := \text{result}_0 \quad A_s = 22114 \text{ mm}^2$$

$$a := \text{result}_1 \quad a = 43.708 \text{ mm}$$

### Top Reinforcement Y-Y

$$\text{Pile}_t := \frac{P}{n} - \frac{M_{yy} \cdot x_{\text{offset}}}{I_{yy}} \quad \text{Pile}_t = 10411 \text{ kN}$$

$$cs := \frac{1}{2} l_{\text{coeff}} \quad cs = 1 \text{ m}$$

$$x_{cs} := \frac{1}{2} l_{ftg} - cs \quad x_{cs} = 4 \text{ m}$$

$$M_{ftg} := -\frac{1}{2} x_{cs}^2 \cdot w_{ftg} \cdot d_{ftg} \cdot \rho_{\text{conc}} \quad M_{ftg} = -5000 \text{ kN}\cdot\text{m}$$

$$M_{soil} := -\frac{1}{2} x_{cs}^2 \cdot w_{ftg} \cdot d_{soil} \cdot \rho_{\text{soil}} \quad M_{soil} = -240 \text{ kN}\cdot\text{m}$$

$$M_{\text{pile}} := 3 \cdot \text{Pile}_t \cdot (x_{\text{offset}} - cs) \quad M_{\text{pile}} = 103069 \text{ kN}\cdot\text{m}$$

$$M_u := (M_{pile} + M_{ftg} + M_{soil})$$

$$M_u = 97829 \text{ kN}\cdot\text{m}$$

$$M_u := \text{if}(M_u > 0.0 \text{ kN}\cdot\text{m}, 0.0 \text{ kN}\cdot\text{m}, M_u)$$

$$M_u = 0 \text{ kN}\cdot\text{m}$$

$$\phi = 0.9$$

### Solver Initial Guess

$$A_s := 10000 \cdot \text{mm}^2$$

$$a := 1000 \text{ mm}$$

$$b := w_{ftg}$$

$$d := d_{ftg} - cl_{top}$$

Given

$$a = \frac{A_s \cdot f_y}{0.85 \cdot f_c \cdot b}$$

$$M_u = \left[ A_s \cdot f_y \cdot \left( d - \frac{a}{2} \right) \right] \cdot \phi$$

$$\text{result} := \text{Find}(A_s, a)$$

<~~~~~ Solve for Area of Steel, and dimension a

$$A_s := \text{result}_0 \quad A_s = 0 \text{ mm}^2$$

$$a := \text{result}_1 \quad a = -9.802 \times 10^{-4} \text{ mm}$$

### Temperature Requirements

$$\frac{1}{8} \cdot \text{in}^2 \cdot \frac{w_{ftg}}{\text{ft}} = 2646 \text{ mm}^2$$

### Minimum Reinforcement - 1.2 Mcr

$$A_s := 1000 \text{ mm}^2 \quad \text{Guess}$$

$$d := d_{ftg} - cl_{top} \quad d = 2.425 \text{ m}$$

$$S := \frac{w_{ftg} \cdot d_{ftg}^2}{12} \quad S = 5.208 \text{ m}^3$$

$$f_r := 7.5 \cdot \sqrt{f_c \cdot \text{psi}}$$

$$M_u := 1.2 \cdot S \cdot f_r \quad M_u = 1.946 \times 10^4 \text{ kN}\cdot\text{m}$$

Given

$$a = \frac{A_s \cdot f_y}{0.85 \cdot f_c \cdot b}$$

$$M_u = \left[ A_s \cdot f_y \cdot \left( d - \frac{a}{2} \right) \right] \cdot \phi$$

result := Find( $A_s$ , a) <~~~~~ Solve for Area of Steel, and dimension a

$$A_s := \text{result}_0 \quad A_s = 21418 \text{ mm}^2 \quad <~~~~~ \text{Controls}$$

$$a := \text{result}_1 \quad a = 42.332 \text{ mm}$$

### One Way Shear

$$\phi := 0.85$$

**X-X**

$$C := \frac{P}{n} + \frac{M_{xx} \cdot y_{\text{offset}}}{I_{xx}} \quad C = 16225 \text{ kN}$$

$$T := \frac{P}{n} - \frac{M_{xx} \cdot y_{\text{offset}}}{I_{xx}} \quad T = 10411 \text{ kN}$$

Critical Section

$$cs := \frac{1}{2} \cdot w_{\text{coeff}} + d_{ftg} \quad cs = 3 \text{ m}$$

$$y_{cs} := \frac{1}{2} \cdot w_{ftg} - cs \quad y_{cs} = 2 \text{ m}$$

$$V_u := 3 \cdot \text{piles} \cdot C - y_{cs} \cdot l_{ftg} \cdot (d_{ftg} \cdot \rho_{conc} + d_{soil} \cdot \rho_{soil}) \quad V_u = 47365 \text{ kN}$$

$$V_c := 2 \cdot \sqrt{f_c \cdot \text{psi}} \cdot l_{ftg} \cdot (d_{ftg} - \text{clr}_{bot}) \quad V_c = 19513 \text{ kN}$$

$$V_s := \frac{V_u}{\phi} - V_c \quad V_s = 36210 \text{ kN}$$

$s := 300 \text{ mm}$       Assumed

$$A_v := \frac{V_s \cdot s}{(d_{ftg} - \text{clr}_{bot}) \cdot f_y} \quad A_v = 11006 \text{ mm}^2 < \sim \sim \sim \text{Controls}$$

$$V_u := 3 \cdot \text{piles} \cdot T + y_{cs} \cdot l_{ftg} \cdot (d_{ftg} \cdot \rho_{conc} + d_{soil} \cdot \rho_{soil}) \quad V_u = 3.254 \times 10^4 \text{ kN}$$

$$V_c := 2 \cdot \sqrt{f_c \cdot \text{psi}} \cdot l_{ftg} \cdot (d_{ftg} - \text{clr}_{bot}) \quad V_c = 19513 \text{ kN}$$

$$V_s := \frac{V_u}{\phi} - V_c \quad V_s = 18773 \text{ kN}$$

$$A_v := \frac{V_s \cdot s}{(d_{ftg} - \text{clr}_{bot}) \cdot f_y} \quad A_v = 5706 \text{ mm}^2$$

### Y-Y

$$C := \frac{P}{n} + \frac{M_{yy} \cdot x_{offset}}{I_{yy}} \quad C = 16225 \text{ kN}$$

$$T := \frac{P}{n} - \frac{M_{yy} \cdot x_{offset}}{I_{yy}} \quad T = 10411 \text{ kN}$$

### Critical Section

$$cs := \frac{1}{2} \cdot l_{coff} + d_{ftg} \quad cs = 3.5 \text{ m}$$

$$x_{cs} := \frac{1}{2} \cdot l_{ftg} - cs \quad x_{cs} = 1.5 \text{ m}$$

### Pile Compression Case

$$V_u := 3 \cdot \text{piles} \cdot C - x_{cs} \cdot l_{ftg} \cdot (d_{ftg} \cdot \rho_{conc} + d_{soil} \cdot \rho_{soil}) \quad V_u = 4.769 \times 10^4 \text{ kN}$$

$$V_c := 2 \cdot \sqrt{f_c \cdot \text{psi}} \cdot l_{ftg} \cdot (d_{ftg} - \text{clr}_{bot}) \quad V_c = 19513 \text{ kN}$$

$$V_s := \frac{V_u}{\phi} - V_c \quad V_s = 36596 \text{ kN}$$

s := 300mm      Assumed

$$A_v := \frac{V_s \cdot s}{(d_{ftg} - \text{clr}_{bot}) \cdot f_y} \quad A_v = 11123 \text{ mm}^2 < \sim \sim \sim \text{Controls}$$

### Pile Tension Case

$$V_u := 3 \cdot \text{piles} \cdot T + x_{cs} \cdot l_{ftg} \cdot (d_{ftg} \cdot \rho_{conc} + d_{soil} \cdot \rho_{soil}) \quad V_u = 3.222 \times 10^4 \text{ kN}$$

$$V_c := 2 \cdot \sqrt{f_c \cdot \text{psi}} \cdot l_{ftg} \cdot (d_{ftg} - \text{clr}_{top}) \quad V_c = 20136 \text{ kN}$$

$$V_s := \frac{V_u}{\phi} - V_c \quad V_s = 17765 \text{ kN}$$

$$A_v := \frac{V_s \cdot s}{(d_{ftg} - \text{clr}_{top}) \cdot f_y} \quad A_v = 5233 \text{ mm}^2$$

### Punching Shear

$\phi = 0.85$

P = 106544 kN      Column Axial + Soil + Footing

$$c_{sy} := \frac{1}{2} \cdot (d_{ftg} + l_{colevf}) \quad c_{sy} = 2.25 \text{ m}$$

$$c_{sx} := \frac{1}{2} \cdot (d_{ftg} + w_{colevf}) \quad c_{sx} = 1.75 \text{ m}$$

$$b_o := 2 \cdot (c_{sx} + c_{sy}) \quad b_o = 8 \text{ m}$$

$$\text{area} := (w_{ftg} \cdot l_{ftg}) - 2 \cdot c_{sx} \cdot c_{sy}$$

$$V_u := \left[ P - \text{area} \cdot (d_{soil} \cdot \rho_{soil} + d_{ftg} \cdot \rho_{conc}) \right] \cdot \frac{1}{\phi} \quad V_u = 118247 \text{ kN}$$

$$\beta_c := \frac{\min(w_{ftg}, l_{ftg})}{\max(w_{ftg}, l_{ftg})}$$

$$\text{factor} := \min\left[\left(2 + \frac{4}{\beta_c}\right), 4\right]$$

$$\text{factor} = 4$$

$$V_c := \text{factor} \cdot \sqrt{f_c \cdot \text{psi}} \cdot b_o \cdot (d_{ftg} - \text{clr}_{bot})$$

$$V_c = 31221 \text{ kN}$$

$$V_s := V_u - V_c$$

$$V_s = 87026 \text{ kN}$$

$$s := 300 \text{ mm} \quad \text{Assumed}$$

$$A_v := \frac{V_s \cdot s}{(d_{ftg} - \text{clr}_{bot}) \cdot f_y}$$

$$A_v = 26452 \text{ mm}^2$$